A Junior College in the First MCM

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The Paper

The beauty of the MCM is that students experience government- or industrial-type problem-solving while writing a paper on a weekend at their own campus.

I include below parts of the paper presented in the first MCM by Jody Baker, Jeanine Scheeden, and Lynda Ferguson of Southern Seminary Junior College. They were responding to the problem titled "Modeling an Animal Population," in which they were instructed to select a fish or mammal and an environment, and then to formulate an optimal policy for harvesting members of the population. This team chose the burros in the Grand Canyon of the U.S.

> Start with 200 burros, with one-half being female (100 female). Their ages range from 1 to 8 years. Those one year old and under are nonbreedable. One-half of the eight-year-olds are nonbreedable. The result is a breeding herd of 87 females.
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> Each female has an average of three offspring per lifetime. This was deduced by taking into consideration that the burros would not breed until two years of age; therefore, there would not be a foal until the third year. Also taken into account is the fact that some may be aborted, or stillborn, or the burro may not have conceived in a particular year. If each burro lives approximately eight years, then every year one-eighth are dying. This is 25 burros, half of which would be female.

As their work progressed, the team discovered the differential equations necessary for the rate-of-growth problem [Marcus-Roberts 1983, 46]:

$$\frac{dp(t)}{dt} = ap(t),$$

where a is the rate of increase of the herd. Taking the logarithm of each side, they got

$$p(t) = p_0 e^{a(t-t_0)}.$$

They modeled and then computed their solution on the 1,000 existing animals, considering death rate and birth rate with the parameters of population stability in a deteriorating vegetation situation. This leads to the conclusion that to maintain the herd at 1,000, they must have

$$p(t) = 1000e^{a(0.25\times 1.0)} = 1295.$$

Thus, 295 burros per year must be harvested for adoption or other park uses.

The contest accomplished for this team exactly what it was designed to do. These students were starting their second semester of calculus, and they extended themselves by studying material ahead in their text [Thomas and Finney 1968]. For example, they learned how to make use of partial fractions to integrate a rational expression. They researched and understood the mathematics that they used from other sources. They learned what the modeling process can accomplish, and they had the experience of a team effort.

The Team

Dr. Ben Fusaro, the founding director of MCM, says that "It is not the three best that make the team, but the best three." The team needs the skills of drafting, typing, and computer programming, plus logic and mathematical maturity and the ability to do research and presentation with style.

There are several ways to choose the team. One is to look for students in the highest-level courses and choose the best combination for the team, making it an honor to be chosen. Another is to announce the contest and invite volunteers. One can also recruit students who are computer-oriented and who appreciate mathematical applications, or choose students who work well together and have productive results.

The team must be trained. Generally, students have not learned how to write a mathematical research paper. They can develop this skill by reading published papers, including published Outstanding papers from previous contests, or by writing papers and having them evaluated. Since at most two teams can enter from any department, it might be good to have a junior team and a senior team. This would allow for experienced team members to move up from the previous year's team.

The Advisor

The advisor is analogous to a coach. He or she must recruit and choose the students, arrange for team registration for the contest, secure a room and appropriate computing equipment, and do pre- and post-competition press releases. The advisor must plan a time schedule with the team, practice, and then give support and encouragement during the contest.

The rewards of this coaching experience are many:

- You will have an intellectual experience with a group of students outside of the classroom.
- You will be respected not only by the team members but also by the academic community.
- You will be helping students learn the art of mathematical modeling.
- The knowledge that you gain of industrial problems and modeling solutions will lead you to better classroom teaching.
- You will be giving your students a first-class ticket into an interview with any employer or applied science department.

On a two-year campus, as well as at four-year schools, there are students who have superior qualities that fit the needs of the team. It is the advisor's job to find those students and help them take advantage of this fantastic opportunity.

References

Marcus-Roberts, Helen. 1983. A comparison of some deterministic and stochastic models of population growth. 1985. Chapter 3 in *Life Science Models*, edited by Helen Marcus-Roberts and Maynard Thompson. Vol. 4 of *Modules in Applied Mathematics*, edited by William F. Lucas. New York: Springer-Verlag.

Thomas, George, and Ross Finney. 1968. *Calculus and Analytic Geometry.* 5th ed. Reading, MA: Addison-Wesley.

About the Author

Alice Williams has taught at high-school, junior-college, and university levels. She has researched at University College, Oxford University, in the area of mathematical modeling. She lives with her husband in Lexington, VA.